Physics of Freefall

A mass will feel the force of gravity - this force will encourage the mass to accelerate downwards. The acceleration and final velocity will depend on the gravitational field strength, -9.81 ms^{-1} at the earth's surface, and drag if it is not moving through a vacuum. You will be familiar with acceleration due to gravity however it is worth revisiting the ideas.

What is Mass?

It appears that there are *two* distinct forms of mass, gravitational and inertial.

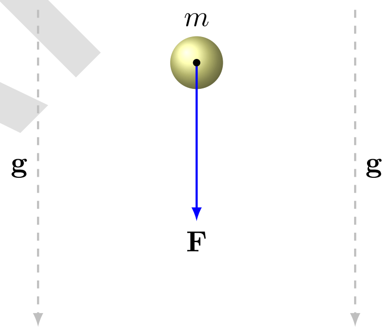
Gravitational Mass

A mass m in a uniform gravitational field experiences a force due to gravity given by

$$\mathbf{F} = m\mathbf{g}$$

from Newton's 2nd law of $\mathbf{F} = m\mathbf{a}$

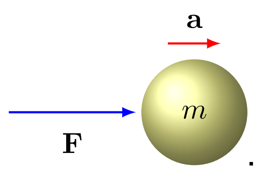
$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{m\mathbf{g}}{m} = \mathbf{g}$$



Thus we have that in a uniform gravitational field $\mathbf{a} = \mathbf{g}$.
All masses will fall at the same rate in a particular gravitational field. Intuitively one can think of this as - twice the mass equal twice the force – but this force has to accelerate twice the mass.
So is mass really a gravitational charge, a property of an object which both creates a surrounding gravitational field and experiences a force in a gravitational field?

Inertial Mass

However, you have seen Newton's 2nd law $\mathbf{F} = m\mathbf{a}$ giving us $m = \frac{\mathbf{F}}{\mathbf{a}}$. Here m is the inertial mass. Inertia is that property of a body which requires a force to accelerate it. Inertial mass exists whether or not the object is in a gravitational field. In classical (non-relativistic) physics inertial mass appears utterly distinct from gravitational mass.



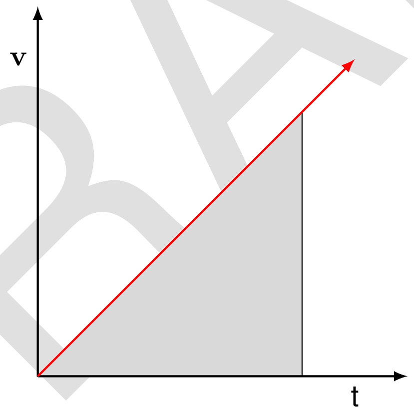
Are they the same?

So we have two types of mass - mass as the amount of *gravitational charge* and mass being a measure of *inertia*. Are these two masses the same? Both Newton and Einstein believed so and experiments have so far failed to find a case where inertial mass is not directly proportional to gravitational mass with a precision of about $1 : 10^{13}$. There is a Nobel prize going if you can prove they differ.
This apparent equivalence of inertial and gravitational mass is a foundation stone of Newton's Principia Mathematica and Einstein's General Theory of Relativity - where it is known as the *Weak Equivalence Principle*.

Freefall without Drag

At the earth's surface $\mathbf{g}$ is about 10 metres per second per second ms^{-2}. What does this mean? Starting from rest, after 1 seconds a mass will be moving at 10ms^{-1}, after 2 sec 20ms^{-1} and so on.

t sec	0	1	2	3	4	5	6
v ms^{-1}	0	10	20	30	40	50	60

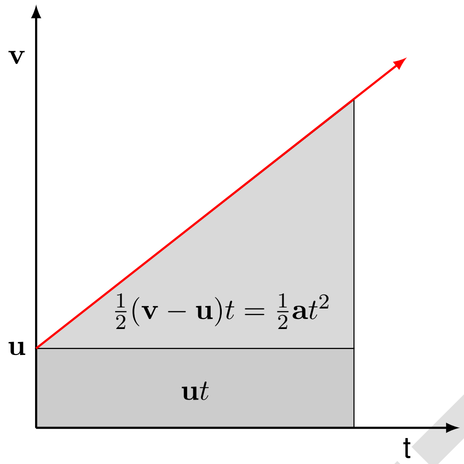


area = $\int \mathbf{v}dt$ = displacement

In this simple case we can see that $\mathbf{v} = \mathbf{a}t$ and the displacement $\mathbf{s} = \frac{1}{2}\mathbf{v}t$. Sustituting in for the velocity gives $\mathbf{s} = \frac{1}{2}\mathbf{v}t = \frac{1}{2}\mathbf{a}t^2$.

If we start with an initial velocity $\mathbf{u}$, say 15 ms^{-1} we have:-

t sec	0	1	2	3	4	5	6
v ms^{-1}	15	25	35	45	55	65	75

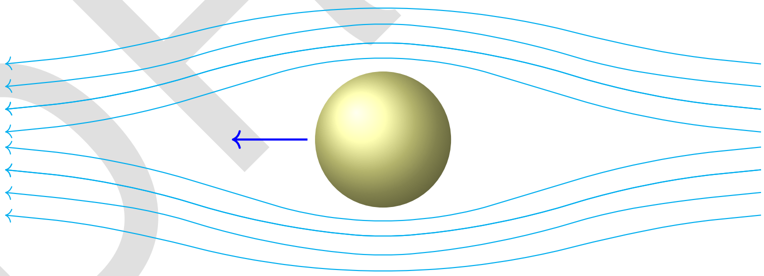


area $\mathbf{s} = \mathbf{u}t + \frac{1}{2}(\mathbf{v}-\mathbf{u})t = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

But what about friction?

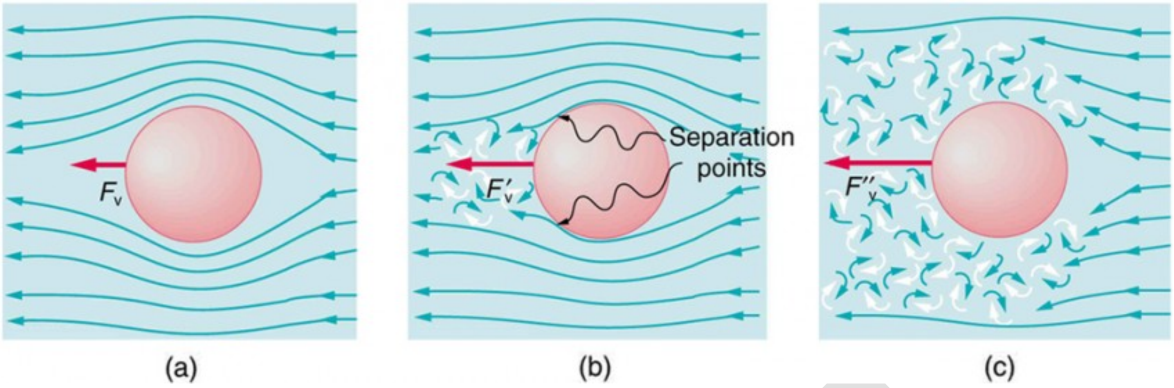
Freefall with Drag

A body moving within a fluid experiences friction called drag. Its magnitude depends upon the body's size, velocity, surface, and shape as well as the fluid through which it moves.
The problem is that drag is hideously complex as well as terribly important. There must be a shape for a plane, car, submarine which will minimise drag and thus maximise speed and fuel efficiency. Unfortunately since we cannot definitively solve the equations for drag we cannot calculate these perfect shapes.



low-speed .. laminal flow around a sphere

At low speeds the fluid flows smoothly around the object creating a constant drag force. As the speed increases the flow becomes more complex as the system moves from laminar to turbulent flow. Here vortices and eddies form and break away from the surface in a non-linear manner leading to drag which is difficult to predict.



Stokes' Law - an approximation

Georges Stokes in 1851 derived a simple expression for the drag on a small sphere moving slowly through a viscous fluid- ie. one where the flow is essentially laminar

$$\mathbf{F_d} = 6\pi r\eta\mathbf{v}$$

where

- F_d is the drag force
- r is the sphere's radius
- η is the liquid's *dynamic viscosity* measure in Pascal seconds Pa s
- $\mathbf{v}$ is the sphere's velocity relative to the liquid

For air at room temperature η is approx 18x10^{-6} Pa whilst water is about 1x10^{-3} Pa. In a general simulation I would experiment with a much higher figure.

References

[1] Wikipedia. *https://en.wikipedia.org/wiki/Equivalence_principle*

[2] Markus Pössel. *https://www.einstein-online.info/en/spotlight/inertial-and-gravitational-mass/*

[3] MIT OpenCourseWare. *https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-090-introduction-to-fluid-motions-sediment-transport-and-current-generated-sedimentary-structures-fall-2006/course-textbook/ch3.pdf*